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121. Proposed by AUGUSTUS J. REEF, Student in Illinois State Normal University, Carbondale, Ill.

Construct a triangle having given its three medians. [From Wentworth's *Plane and Solid Geometry*.]

I. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.; ALOIS F. KOVARIK, Instructor in Mathematics, Decorah Institute, Decorah, Ia.; CHAS. C. CROSS, Whaleyville, Va.; and the PROPOSER.

*Each median intersects the other medians at a common point two-thirds of the distance from the vertex to the middle of the opposite side.*

Let  $AF$ ,  $BD$ , and  $CE$  be the three medians of a triangle.

*Trisect each of the medians.*

Take any point  $O$  as a center, and with a radius equal to two-thirds of  $CE$ , the greatest median, describe the semi-circumference  $HCG$ .

Draw the diameter  $HOG$ .

With a radius equal to two-thirds of  $BD$ , the next largest median, and  $O$  as a center, intersect  $HOG$  at  $B$ .

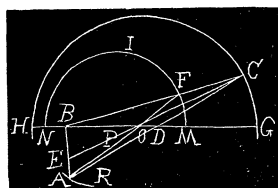
Bisect, respectively,  $HB$  at  $N$ ,  $BG$  at  $M$ , and  $MN$  at  $P$ .

Then, with  $NP$ ,  $=PM$ , as a radius and  $P$  as a center, draw the indefinite arc  $NIFM$ . This arc bisects any straight line drawn from point  $B$  to outer arc.

With a radius equal to one-third of median  $AF$  and  $O$  as a center, intersect arc  $NIFM$  at  $F$ ; and with a radius equal to two-thirds of  $AF$  and  $O$  as a center, describe the indefinite arc  $AR$ .

Through  $F$  and  $O$  draw line  $FA$  terminating in arc  $AR$ . Also draw lines  $BA$  and  $AC$ .

Then will  $ABC$  be the required triangle.



II. Solution by HENRY HEATON, M. Sc., Atlantic, Ia.; ELMER SCHUYLER, Reading, Pa.; J. D. CRAIG, A. B., New Germantown, Pa.; J. SCHEFFER, A. M., Hagerstown, Md.; P. S. BERG, B. Sc., Principal of Schools, Larimore, N. D.; COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.; GAYLOR CAMERON, Tiffin, O.; and W. H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.

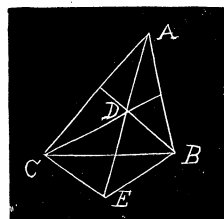
Construct the triangle  $DEB$  such that  $DE$ ,  $DB$ , and  $EB$  shall be, respectively, equal to two-thirds the given medians from the angles  $A$ ,  $B$ , and  $C$ , of the required triangle.

Draw  $EC$  parallel to  $BD$ , and  $DC$  parallel to  $BE$ , meeting in  $C$ .

Prolong  $ED$  to  $A$ , making  $DA=ED$ . Join  $AB$  and  $AC$ .

Then will  $ABC$  be the the required triangle.

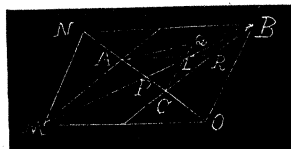
The demonstration is obvious.



III. Solution by J. W. YOUNG, Columbus, Ohio, and G. B. M. ZERE, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Construct a parallelogram such that  $MN$ ,  $MO$ , and  $MB$  shall be double the given medians.

Draw the other diagonal  $NO$ . Trisect  $NO$  in  $A, C$ . The triangle  $ABC$  is the one required, since  $PB$  is evidently one of the medians given, and the other medians  $QC$  and  $AR$  are, respectively, equal to  $\frac{1}{2}OB$  and  $\frac{1}{2}NB$ . This is clear, from the considerations of the similar triangles  $AOB$  and  $AQC$  ( $AQ=\frac{1}{2}AB$ ,  $AC=\frac{1}{2}AO$ ,  $\therefore QC=\frac{1}{2}OB$ ), and  $NCB$  and  $ACR$  ( $AC=\frac{1}{2}NC$ ,  $RC=\frac{1}{2}BC$ ,  $\therefore AR=\frac{1}{2}NB$ ).



## CALCULUS.

90. Proposed by ELMER SCHUYLER, Reading, Pa.

Prove that the evolute of the logarithmic spiral is an equal logarithmic spiral.  
[From Byerly's *Integral Calculus*.]

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; GEORGE LILLEY, Ph. D., L. L. D., Professor of Mathematics, State University, Eugene, Ore.; WALTER H. DRANE, A. M., Graduate Student, Harvard University, Cambridge, Mass.; and ELMER SCHUYLER, Reading, Pa.

The intrinsic equation to the logarithmic spiral  $s=k(c'-1)$ .

$ds/dt = kc' \log c$ , for the evolute  $\sigma = \pm (ds/dt) \Big|_0^t$ .

$\therefore \sigma = kc' \log c - k \log c = k \log c (c' - t)$ .

$\therefore \sigma = k'(c' - 1)$ , an equal spiral.

II. Solution by J. SCHEFFER, A. M., Hagerstown, Md.; and COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.

Let  $P$  be a point of the given curve  $r=a^\theta$ ,  $O$  the center of curvature,  $PQ$  a tangent at  $P$ ,  $PO=\rho$ ,  $SQ=p$ —the perpendicular from  $S$  upon  $PQ$ ,  $SP=r$ ,  $SO=r'$ ,  $SM$  perpendicular to  $OP$  and  $=p'$ .

The pedal equation of the given curve  $r=a^\theta$  is  $r=p\sqrt{1+(\log a)^2}$ ; we also have  $r'^2=\rho^2+r^2-2\rho p$ , but  $\rho=r\sqrt{1+(\log a)^2}$ .

$\therefore r'=r \log a$ , and since  $p'^2=r^2-p^2$ , we have

$$p'^2 = \frac{r^2(\log a)^2}{1+(\log a)^2} \quad \therefore p' = \frac{r \log a}{\sqrt{1+(\log a)^2}} \quad \therefore p' = \frac{r'}{\sqrt{1+(\log a)^2}},$$

or,  $r'=p'\sqrt{1+(\log a)^2}$ , which is the pedal equation of the evolute and exactly like the pedal equation of the logarithmic spiral.

III. Solution by CHAS. E. MYERS, Canton, Ohio; and P. H. PHILBRICK, M. S., C. E., Chief Engineer for Kansas City, Watkins & Gulf Railway Co., Lake Charles, La.

Let  $r$ —the radius vector of the given curve,  $p$ —the perpendicular on the tangent,  $r_1$ —the radius vector of the evolute,  $p_1$ —the perpendicular on its tangent, and  $R$ —the radius of curvature.

We have for the curve,  $r=cp \dots \dots (1)$ .